Function Limits

44. Calculate the limit:

$$\lim_{x\to 1} \frac{x^2+x-2}{-2x^2+6x-4},$$

without using L'Hôpital's rule (which we studied in Analysis II).

45. Calculate the limit:

$$\lim_{x\to+\infty} x \operatorname{arcctg} x,$$

without using L'Hôpital's rule (which we studied/will study in Analysis II).

46. Calculate the limit

$$\lim_{\alpha\to\frac{\pi}{2}-0} \left(\sqrt{\operatorname{tg}^2\alpha + \frac{1}{\cos\alpha}} - \operatorname{tg}\alpha \right)$$

of course, calculate it **without** using L'Hôpital's rule (which we have covered/will cover in Analysis II).

Series

47. Using partial fraction decomposition, sum the series

$$\sum_{n=1}^{\infty} \frac{1}{(n+7)(n+8)}.$$

48. Determine precisely for which real numbers $a \in \mathbb{R}$ the series

$$\sum_{n=1}^{+\infty} \frac{a\, n^5}{2^n}$$

converges (if such a number a exists).

49. Determine precisely for which real numbers $\alpha \in \mathbb{R}$ the series

$$\sum_{n=1}^{+\infty} \frac{\alpha}{\sqrt{n}}$$

converges (if such a number α exists).

50. Determine whether the given series converges or not

$$\sum_{n=1}^{\infty} \frac{1}{n(\sqrt{7}-2)^n}.$$

51. Determine whether the given series converges or not

$$\sum_{n=1}^{\infty} n! \left(\frac{2}{n}\right)^n.$$

All above math problems are taken from the following website:
`https://osebje.famnit.upr.si/~penjic/teaching.html`.
The reader can find all solutions to the given problems on the same page.